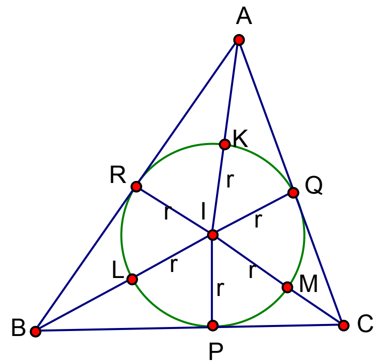
3872.(**Corrected**) **Proposed by F. R. Ataev**.

Let x, y, z be the distances from the vertices of a triangle to its incircle and let r be the inradius of the triangle. Show that the area of the triangle is given by

$$A = \frac{\sqrt{xyz(x+2r)(y+2r)(z+2r)}}{r}.$$

Solution by Arkady Alt , **San Jose** ,**California**, **USA**.



Let I be incenter of a triangle ABC and $K,\ L,\ M$ be, respectively, points of intersection of segments AI, BI, CI with incircle. Let P, Q, R be, respectively, points of tangency of incircle with sides BC, CA, AB.

Then $x = AK, y = BL, z = CM$ and denoting $u := AR = AQ, v := BR = BP$, $w := CP = CQ$ we obtain (by Tangent Secant Theorem)

$(x+2r)x = u^2, (y+2r)y = v^2, (z+2r)z = w^2$.

Let s be semiperimeter of $\triangle ABC$. Then $u = s-a, v = s-b, w = s-c, u+v+w = s$ and $A = sr = (u+v+w)r$.

Since by Heron's formula $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{(u+v+v)uvw}$ then

$$A^4 = (u+v+v)^2u^2v^2w^2 = \frac{A^2(x+2r)x(y+2r)y(z+2r)z}{r^2} \iff$$

$$A^2 = \frac{xyz(x+2r)(y+2r)(z+2r)}{r^2} \iff A = \frac{\sqrt{xyz(x+2r)(y+2r)(z+2r)}}{r}.$$